1 Basic information

The BILAN model (version 1.7) has been developed for assessing water balance components of a catchment using a monthly step. The structure of the model is formed by a system of relationships describing basic principles of the water balance on the land surface, the zone of aeration, including the effect of vegetation cover, and groundwater. Air temperature is used as an indicator of energy conditions, which affect significantly the water balance components.

1.1 Entry data

The entry data of the model are monthly series of catchment precipitation and air temperature. Furthermore, relative air humidity or potential evapotranspiration series are required. Monthly runoff series at the outlet from the catchment are used to calibrate model parameters.

1.2 Potential evapotranspiration

The potential evapotranspiration can be either read (instead of the relative air humidity series, option 2) from the input file or it can be calculated from the saturation deficit (option 1) by
using functions (provided as tables) that have been derived for individual months and for
different bioclimatic zones from empirical graphs (representing conditions of the Northern
Hemisphere) given by Rekomendatsii (1976). The saturation deficit (in mbar) is calculated
from data on the air temperature and relative air humidity.

The following bioclimatic zones are included in the model:

a) tundra;
b) coniferous forest;
c) mixed forest;
d) deciduous forest, and
e) steppe.

Each bioclimatic zone is characterised by a characteristic mean air temperature. The model
has an interpolation algorithm, which uses catchment long-term average air temperature for
interpolating between the bioclimatic zones that is between particular tables.

1.3 Simulated series
The model simulates for a catchment monthly series of potential evapotranspiration (option 1),
actual evaporation, infiltration into the zone of aeration, percolation of water towards the
groundwater aquifer, and water storage in the snow cover, zone of aeration (soil) and
groundwater aquifer. The total runoff consists of three components, which are direct runoff,
through flow (interflow) and base flow.

1.4 Model parameters
The model has eight free model parameters and uses an optimisation algorithm for their
calibration using gauged streamflow. The optimisation aims at attaining the best fit between the
observed and simulated runoff series, for which several optimisation criteria are available.

2 Model description
The internal structure of the model is given in Figure 1 and individual algorithms are described
in the text below.
2.1 Type of regime
Some of the model algorithms are applied dependent on conditions in a particular month. Using mean monthly air temperature, the model distinguishes between summer conditions and winter conditions. Summer conditions are assumed if the temperature:

\[ t(i) \geq 0. \]  \hspace{1cm} (1)

where \( t \) is mean monthly air temperature and \( i \) is month number.

Algorithms for winter conditions are used, if mean monthly air temperature is negative. A snow melting algorithm is used instead of the summer algorithm when there is a snow cover in the catchment.

2.2 Components of total runoff
The model simulates total runoff \( rm(i) \) as the sum of three components:

\[ rm(i) = dr(i) + I(i) + bf(i) \]  \hspace{1cm} (2)

where \( dr(i) \), \( I(i) \) and \( bf(i) \) are direct runoff, through flow (interflow) and base flow, respectively.
The $dr(i)$ component of the total runoff (summer direct runoff caused by a high-intensity rain) is considered to be the fast flow component, which is not available for evaporation and which does not affect the soil water balance. Irrespective of the season, the through flow (interflow) $I(i)$ results from the water balance as excess water in the zone of aeration in the considered month. This runoff component is assumed to include also direct runoff if it occurs in winter or during the period when snow melts.

The slow component of total runoff is base flow $bf(i)$, whose delay time in the basin is longer than one month. It is generated by outflow from the groundwater storage.

### 2.3 Formation of direct runoff under summer conditions

Direct runoff occurring during the summer season is caused by rainfall with high intensity. It is calculated as:

$$dr(i) = Alf \cdot p(i)^2 \cdot \left(\frac{ss(i-1)}{Spa}\right)$$  \hspace{1cm} (3)

where $Alf$ is a parameter of the quadratic relationship between direct runoff and rainfall, $p(i)$ is precipitation in month $i$, $ss(i-1)$ is soil moisture in month $i-1$, and $Spa$ is a parameter expressing soil moisture capacity.

The infiltration ($inf$) into the soils equals:

$$inf(i) = p(i) - dr(i)$$  \hspace{1cm} (4)

The infiltration is input for the zone of aeration.

### 2.4 Evaporation and soil water balance under summer conditions

If precipitation reduced by direct runoff, $inf(i)$, calculated by Equation 4 equals or exceeds potential evapotranspiration then the catchment evapotranspiration is equal to the potential evapotranspiration:

$$e(i) = pe(i) \quad \text{if} \quad inf(i) \geq pe(i)$$  \hspace{1cm} (5)

Excess water $inf(i)-pe(i)$ is available to feed soil moisture:

$$ss(i) = ss(i-1) + inf(i) - pe(i)$$  \hspace{1cm} (6)

If capacity of the soil moisture storage is exceeded, the remaining water percolates downwards:

$$perc(i) = ss(i) - Spa \quad \text{if} \quad ss(i) > Spa$$  \hspace{1cm} (7)

Then the soil moisture storage $ss(i)$ is equal to $Spa$ parameter:
If potential evapotranspiration exceeds the precipitation reduced by direct runoff, the catchment evapotranspiration is supplied from the soil moisture storage, which is being depleted:

$$ss(i) = ss(i-1) \exp((\text{inf}(i) - \text{pe}(i))/\text{Spa}) \quad \text{if} \quad \text{inf}(i) < \text{pe}(i)$$

(9)

The catchment evapotranspiration equals the sum of infiltration and the soil moisture depletion:

$$e(i) = \text{inf}(i) + ss(i-1) - ss(i)$$

(10)

No water is available for percolation.

2.5 Evaporation and water balance on the land surface under winter conditions and during the period when snow melts

If the sum of precipitation and water storage in the snow cover in the considered month exceeds potential evapotranspiration, it is assumed that the catchment evapotranspiration is equal to the potential evapotranspiration:

$$e(i) = \text{pe}(i).$$

(11)

The quantity of the remaining water, which is potentially available for infiltration (available water in the form of snow), is:

$$akt(i) = sw(i-1) + p(i) - \text{pe}(i)$$

(12)

where $sw(i-1)$ is water storage in the snow cover in month $i-1$.

However, the virtual quantity of water available for infiltration is limited by the heat capacity of the air to melt the snow cover in the considered month, which is, under snow melting conditions ($t(i) \geq 0$), expressed as:

$$pot(i) = t(i) \cdot Dgm + p(i)$$

(13)

where $Dgm$ is a parameter expressing the rate of snowmelt dependent on the air temperature.

Under winter conditions, part of precipitation is assumed to be rainfall (liquid water) and/or the existing snow cover melts partially. This is supposed to occur if the monthly air temperature exceeds a certain threshold, which arbitrarily set at $T_{epk} = -8^\circ C$. The coarse time resolution of one month is the main reason for this assumption.
The amount of water that is available in a liquid form is again determined by using the air temperature:

\[ pot(i) = (t(i) - Tepk) \cdot Dgw \]  

(14)

where the \( Dgw \) parameter controls snowmelt during winter conditions.

If the mean monthly air temperature is below the threshold specified as \( Tepk \), the water balance on the land surface is described by the following equation:

\[ sw(i) = sw(i-1) + p(i) - pe(i) \]  

(15)

Infiltration, \( inf(i) \), is supposed to be zero under these conditions. The difference between precipitation and potential evapotranspiration is added to the snow water storage.

If the available water \( akt(i) \) exceeds the limit \( pot(i) \) both under winter and snow melting conditions, the \( akt(i) \) is distributed into a part that infiltrates, \( inf(i) \), and water that remains on the land surface as the snow cover. The following is, therefore, valid:

\[ inf(i) = pot(i) \]  

(16)

\[ sw(i) = akt(i) - inf(i) \]  

(17)

If the limit \( pot(i) \) exceeds the quantity of the available water, this water is fully available for infiltration:

\[ inf(i) = akt(i) \]  

(18)

Under these circumstances water storage in the snow cover is exhausted.

The \( akt(i) \) can exceptionally be negative, when the sum of precipitation and water storage in the snow cover in the considered month is lower than potential evapotranspiration, and thus:

\[ inf(i) = 0 \]  

(19)

\[ sw(i) = 0 \]  

(20)

\[ e(i) = p(i) + sw(i-1) \]  

(21)

### 2.6 Soil water balance under winter and snow melting conditions

Water calculated as infiltration \( inf(i) \) supplies the soil moisture (or zone of aeration), which is assumed to have a maximum capacity given by the \( Spa \) parameter. If the soil capacity is
exceeded, the excess water, $perc(i)$, percolates downwards to feed groundwater recharge storage and interflow. In other words, if the sum of soil water storage from the preceding month, $ss(i-1)$, and infiltration in the given month, $inf(i)$, exceeds the Spa parameter, the following is valid:

$$perc(i) = ss(i-1) + inf(i) - Spa$$  \hspace{1cm} (22) \\
$$ss(i) = Spa.$$  \hspace{1cm} (23) \\
Otherwise:

$$perc(i) = 0,$$  \hspace{1cm} (24) \\
$$ss(i) = ss(i-1) + inf(i)$$  \hspace{1cm} (25)

### 2.7 Distribution of percolation into interflow and groundwater recharge

Percolation $perc(i)$ is divided into through flow (interflow) $I(i)$ that reaches the stream channel in the considered month and recharge $rc(i)$ that replenishes the groundwater storage:

$$I(i) = c \cdot perc(i),$$  \hspace{1cm} (26) \\
$$rc(i) = (1-c) \cdot perc(i)$$  \hspace{1cm} (27)

In the above equations, the $Wic$ parameter is substituted for $c$ under winter conditions, the $Mec$ parameter for snow melting and the $Soc$ parameter in summer.

### 2.8 Groundwater balance and base flow

Groundwater storage $gs(i)$ in month $i$ is calculated as the sum of the storage in the preceding month and recharge $rc(i)$. The base flow represented by the outflow from the groundwater is proportional to groundwater storage at the beginning of the given month and is controlled by the $Grd$ parameter:

$$bf(i) = Grd \cdot gs(i-1)$$  \hspace{1cm} (28) \\
Therefore, the groundwater storage at the end of the month is:

$$gs(i) = rc(i) + (1-Grd) \cdot gs(i-1)$$  \hspace{1cm} (29)

### 3 Optimisation of parameters

In a conventional optimisation procedure, standard error of estimate (standard deviation between the observed and simulated runoff series) would normally be used as an optimisation criterion. A drawback of this criterion is in the fact that its application does not ensure a good fit
between the observed and simulated runoff series in the low flow range. This can substantially be improved by using the sum of relative deviations between the observed and simulated runoff series (‘relative’ means that the deviation in a particular month is divided by the observed runoff) instead of the standard error of estimate. However, this criterion frequently deteriorates the fit in terms of the mean runoff and, therefore, an optimisation procedure combining these two criteria has been developed.

The calibration of the eight model parameters is executed in two steps. In the first step, the standard error of estimate or mean absolute error (mean calculated from absolute deviations between the observed and simulated runoff series, where ‘absolute’ means that negative deviations are converted into positive values) is used as the optimisation criterion to calibrate the Spa, Dgm, Dgw and Alf parameters that affect significantly the mean runoff.

The remaining four parameters (Mec, Wic, Soc, Grd) affecting the runoff distribution into its individual components are then calibrated by using the mean of absolute values of relative deviations. It has been demonstrated by experimental calculations that this calibration procedure ensures mostly an acceptable fit in terms of both mean runoff and low flow runoff, which is predominantly consists of base flow.

**List of symbols and model parameters**

- **i** index of month

**Water balance variables**

- Entry variables – monthly series (totals or mean values)
  - **p** catchment precipitation (mm month\(^{-1}\))
  - **t** air temperature (°C)
  - **h** relative humidity (%)
  - **r** observed runoff (mm month\(^{-1}\))

- Water balance components - monthly totals (all in mm month\(^{-1}\))
  - **pe** potential evapotranspiration
  - **e** catchment evapotranspiration
  - **inf** infiltration into the soil
  - **perc** percolation from the soil
  - **rc** recharge of groundwater storage
  - **I** interflow (through flow)
  - **dr** direct runoff from rainfall
  - **bf** base flow
  - **rm** total runoff (simulated)

**Components of water storage - expressed as columns of water (all in mm)**

- **sw** water storage in snow cover
- soil moisture storage (water storage in zone of aeration)
- groundwater storage

Other variables - monthly totals (all in mm month⁻¹)
- pot water from rainfall or snowmelt (quantity limited by the air temperature)
- akt water in the form of snow available for melting and subsequent infiltration

Model parameters
- Spa capacity of soil moisture storage (mm)
- Alf parameter of rainfall-runoff equation (direct runoff)
- Dgm snow melting factor
- Dgw factor for calculating the quantity of liquid water available on the land surface under winter conditions
- Mec parameter controlling distribution of percolation into interflow and groundwater recharge under conditions of snow melting
- Wic parameter controlling distribution of percolation into interflow and groundwater recharge under winter conditions
- Soc parameter controlling distribution of percolation into interflow and groundwater recharge under summer conditions
- Grd parameter controlling outflow from groundwater storage (base flow)

References